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Stochastic models of the meridional overturning circulation: time scales and patterns of variability

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The global meridional overturning circulation (MOC) varies over a wide range of space and time scales in response to fluctuating ‘weather’ perturbations that may be modelled as stochastic forcing. This study reviews model studies of the effects of climate noise on decadal to centennial MOC variability, on transitions between the MOC regimes and on the dynamics of Dansgaard–Oeschger events characteristic of glacial periods.

Keywords: meridional overturning circulation; stochastic climate models; decadal- to centennial-scale variability; abrupt climate change; Dansgaard–Oeschger events

1. Introduction

The meridional overturning circulation (MOC), which represents oceanic mass transport in the two-dimensional meridional/vertical plane, is a fundamental diagnostic for understanding the role of the ocean in past, present and future climates. In particular, it is believed that sudden changes in the Atlantic MOC (AMOC) are associated with abrupt climate changes prevalent in high-resolution proxy records over the last glacial cycle (e.g. Clark *et al.* 2002; Rahmstorf 2002). The importance of the AMOC to climate lies in its association with much of the total oceanic poleward heat transport in the present-day Atlantic, peaking at approximately 1.2 ± 0.3 PW at 24° N (e.g. Ganachaud & Wunsch 2000). Owing to the importance of the AMOC in the transport of heat to the North Atlantic, the variability and stability of the AMOC (particularly in response to anthropogenic forcing) is a subject of considerable scientific interest (e.g. Wood *et al.* 2003; Meehl *et al.* 2007).

This collection of currents has traditionally been referred to as the thermohaline circulation, a term which is problematic because it emphasizes density gradients while implicitly downplaying the importance of mechanical forcing by the winds and interior turbulent mixing. In fact, ocean density gradients are largely set up by the winds via Ekman currents and surface buoyancy fluxes. Furthermore, mechanical forcing provides the energy source for

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driving these circulations (see Kuhlbrodt *et al.* 2007 for a review), although there is no simple or necessary link between mechanical energy supply and oceanic heat transport (e.g. Saenko & Weaver 2004). Ocean stratification determines the structure and strength of the response to this mechanical driving. Inherent in the use of the terms MOC and AMOC is the fact that wind and buoyancy forcing are inseparable, and that wind and tidal forcing play a fundamental role in providing the energy required for turbulent mixing within the ocean.

The existence and structure of the MOC is fundamentally connected with the locations of deep water formation in the ocean. The two main constituent water masses of the deep North Atlantic Ocean—North Atlantic Deep Water (NADW) at the bottom and Labrador Sea Water at an intermediate level—are currently formed in the Greenland–Iceland–Norwegian Seas and the Labrador Sea, respectively. Deep convection also occurs at a number of locations around Antarctica, but the dense bottom water is susceptible to being trapped by topographic sills (as in the Bransfield Strait) or local circulation patterns (not excluding the Antarctic Circumpolar Current, ACC). In the Southern Ocean, around the southern tip of South America, an enhanced formation of low-salinity Antarctic Intermediate Water (AAIW) also occurs. AAIW plays a critical role in linking the Pacific and Atlantic Oceans and, in particular, in determining the stability of the AMOC (Saenko *et al.* 2003; Weaver *et al.* 2003). Today's climate has no sources of deep water in the North Pacific.

The various forcings that influence (and are in turn influenced by) the MOC (e.g. evaporative and heat fluxes, precipitation, sea-ice advection and melting, wind stresses, ocean eddy activity) display variability over a broad range of space and time scales, with a substantial concentration on time scales of atmospheric variability (subannual to interannual). When studying the dynamics of the MOC on time scales from decades to millennia, it is often convenient to represent these fluctuations as rapidly decorrelating stochastic processes (e.g. Hasselmann 1976). In this paper, the influence of stochastic forcing on the dynamics of the MOC is considered. Section 2 examines the role of atmospheric fluctuations in driving MOC variability on decadal to centennial time scales. The influence of fluctuating forcing on transitions between MOC regimes is discussed in §3. Section 4 addresses the importance of these fluctuations in driving millennial-scale variability during glacial periods. Finally, conclusions are presented in §5.

A fundamental limitation in the study of MOC variability is the paucity of observational evidence. Palaeoclimate data provide evidence of changes in the NADW formation and the AMOC strength on millennial and longer time scales (e.g. Clark *et al.* 2002; McManus *et al.* 2004), while instrumental observations are providing a first characterization of MOC variability on subannual (e.g. Cunningham *et al.* 2007) and interannual to interdecadal time scales (e.g. Bindoff *et al.* 2007). However, direct observations of the MOC of sufficiently high-temporal resolution over sufficiently long periods to address the questions of MOC variability considered in this review are not presently available. This paper consequently focuses primarily on model studies of the MOC and its response to fluctuating forcing. As observational datasets improve, it is to be expected that a more complete comparison of data with model simulations will be possible.

2. Decadal- to centennial-scale variability

Climate variability on time scales ranging from decades to centuries has been identified in the instrumental and observational records of the North Atlantic, associated with a wide range of atmospheric, oceanic and biological processes (e.g. IPCC 2007). While the mechanisms responsible for the low-frequency variability of the climate system are not fully understood, the ocean circulation is believed to play a key role owing to its large thermal inertia. In particular, the variability of the AMOC has received a great deal of theoretical and modelling attention owing to its role as a major transporter of heat to high latitudes in the North Atlantic. In both simple and complex oceanic models, the MOC exhibits variability on decadal and centennial time scales (e.g. Weaver 1995; Bindoff *et al.* 2007), yet the instrumental–observational datasets are generally shorter than 100 years.

Three primary classes of mechanisms have been proposed to explain natural decadal- to centennial-scale variability of the MOC: (i) damped, uncoupled ocean modes excited by atmospheric variability, (ii) unstable, uncoupled ocean modes that express themselves spontaneously, and (iii) unstable or weakly damped, coupled modes of the ocean–atmosphere system. Numerous modelling studies have found self-sustained MOC oscillations associated with diffusive, advective and convective processes (e.g. Weaver & Sarachik 1991; Delworth *et al.* 1993; Weaver *et al.* 1993; Chen & Ghil 1995; Greatbatch & Zhang 1995; Rivin & Tziperman 1997; Fanning & Weaver 1998; Arzel *et al.* 2006). This section considers the major results of those studies which have found that the maintenance of oscillatory variability of the MOC requires the input of energy from fluctuating atmospheric forcing.

Stochastic forcing has traditionally been used to represent high-frequency variability in surface fluxes. If the climate system has no internal oscillatory mode of variability, then the ocean integrates short-term atmospheric fluctuations, transforming the essentially white noise atmospheric forcing into a red response ocean signal (e.g. Hasselmann 1976). However, if the climate system has preferred (if damped) modes of variability, then stochastic forcing results in peaks in the response spectrum at the characteristic time scales of that variability. In general, self-sustained MOC oscillations are due to internal nonlinearities in the climate system, while oscillations driven by stochastic atmospheric forcing can be largely accounted for by linear dynamics and would not exist without the energy from the external stochastic forcing.

In studies of simulated MOC variability on decadal to centennial time scales, stochastic forcing has been used to model fluctuating freshwater fluxes (e.g. Mikolajewicz & Maier-Reimer 1990; Spall 1993; Weaver *et al.* 1993; Weisse *et al.* 1994; Pierce *et al.* 1995; Aeberhardt *et al.* 2000), thermal fluxes (e.g. Griffies & Tziperman 1995; Saravanan & McWilliams 1997; Weaver & Valcke 1998; Kravtsov & Ghil 2004) and surface winds (e.g. Holland *et al.* 2000, 2001; Herbaut *et al.* 2002). A fundamental challenge in the parametrization of these fluctuations by stochastic processes is a lack of knowledge of the forcing fluctuation amplitude, decorrelation time, length scale and distribution. The available observational evidence indicates that the dominant modes of atmospheric variability are essentially white in time but may not be white in space (e.g. Saravanan *et al.* 2000).

Identifying spatial patterns of stochastic forcings that effectively excite oscillations in the modelled AMOC was the focus of studies by Capotondi & Holland (1997), Saravanan & McWilliams (1997) and Tziperman & Ioannou (2002). Analysing the linearized dynamics of a three-box model (representing the polar ocean, the mid-latitude surface ocean and the deep ocean) using generalized stability theory (e.g. Farrell & Ioannou 1996), Tziperman & Ioannou (2002) determined the optimal spatial structure of the noise that results in maximal variance of the AMOC variability. They found that the optimal forcing induces low-frequency variability by exciting salinity variability modes of the AMOC. While a three-box model can be useful for theoretical studies, it is too idealized to be quantitatively accurate. In particular, it is impossible to answer the question of whether observations project onto the predicted optimal modes: more complex models are needed to answer this question. Using a three-dimensional ocean model with idealized basin geometry, Capotondi & Holland (1997) analysed decadal variability by considering the spatial pattern of stochastic forcing as a variable of the problem. The period of oscillation of the simulated AMOC was found to be independent of the spatial pattern, leading to the conclusion that the variability at the decadal time scale is an internal mode of the system and not associated with some characteristics of the forcing (although the amplitude of the response was found to depend on the spatial structure of the forcing). Saravanan & McWilliams (1997) found that spatial resonance, defined as the forcing of a system with a spatial pattern that results in oscillations at a preferred frequency not present in the internal dynamics of the system, was responsible for exciting the oceanic decadal oscillation in a coupled atmosphere–ocean model. Eliminating the spatial correlations in the forcing was found to substantially reduce the variance associated with the interdecadal oscillation of the AMOC. The spatial pattern of the dominant mode of surface heat flux interacted with a single oceanic mode to induce the AMOC oscillations. Other studies have identified spatial patterns of buoyancy flux variations bearing a strong resemblance to the North Atlantic Oscillation (NAO) and which drive multi-decadal to centennial AMOC fluctuations associated with damped internal oscillatory modes of the model ocean (Mikolajewicz & Maier-Reimer 1990; Delworth & Greatbatch 2000; Bentsen *et al.* 2004; Dong & Sutton 2005). On the other hand, the model of Spall (1993) produced sea surface salinity variance as a direct response to the stochastic forcing and not an internal mode of variability. As hypothesized by Hasselmann (1976), Spall found that an undisturbed straightforward integration of the white noise freshwater flux anomalies took place in the Labrador Sea, and it was the irregularly occurring salinity anomalies here that were responsible for the decadal variability in the North Atlantic.

Model studies have also considered the response of the AMOC to fluctuating surface wind stresses. The random wind-forcing field used by Holland *et al.* (2001) was random in time but had a spatial pattern similar to that of observations. The lack of specific time scales in the forcing indicates that the preferred time scales of the model's response were due to internal model physics and not to external forcing. This study showed that the AMOC variability was excited by the stochastic freshwater forcing provided by variable wind-driven Arctic ice export and responded linearly to this forcing at interdecadal time scales. Herbaut *et al.* (2002) described a damped mode of the ocean system requiring stochastic NAO-like wind stress anomalies to maintain the oscillation.

In this study, wind stress forcing drove anomalous currents; the resulting advection of the mean temperature structure generates temperature anomalies that influenced the strength of the simulated AMOC.

Finally, modelling studies have suggested that the character of decadal- to centennial-scale variability in the MOC may be sensitive to the amplitude of the fluctuating forcing. The propagation of simulated salinity anomalies, which mediate the strength of the AMOC, has been shown to be facilitated by larger random freshwater forcing amplitude (e.g. Weaver *et al.* 1993; Skagseth & Mork 1998). Furthermore, changes in the strength of the random forcing have been found to cause transitions between different model oscillatory states (Aeberhardt *et al.* 2000).

Owing to model uncertainties and limited observations, it is not clear whether MOC variability on decadal through centennial time scales is self-sustained or driven by high-frequency variability. Direct comparisons of model results are complicated by variations in model complexity (ranging from box models through two-dimensional models to fully complex three-dimensional models), differences between ocean-only models and coupled ocean-atmosphere models and the variety of methodologies used to identify the dominant ‘modes’ of decadal- to centennial-scale variability. Rivin & Tziperman (1997) suggested that the probability distribution function (pdf) of the MOC time series could be used to differentiate between linear noise-forced and nonlinear self-sustained oscillations. If the pdf is Gaussian when the stochastic forcing is Gaussian, then the oscillations result from the stochastic excitation of damped modes, while for nonlinear oscillations the pdf is strongly non-Gaussian. The refinement of tools such as this to distinguish between driven and self-sustained oscillatory variability will be an important component of determining the importance of fluctuating forcing in producing decadal- to centennial-scale variability in the MOC.

3. AMOC regimes

The present-day AMOC is characterized by a strong NADW formation in the Labrador and Nordic Seas, but both palaeoclimate and modelling studies suggest that the AMOC can exist in other configurations (e.g. Rahmstorf 2002). There are two sets of feedbacks associated with these rearrangements of the AMOC, involving large-scale and local processes, respectively. In its present state, the AMOC transports warm salty water into the North Atlantic, where it is both cooled and freshened. The salt advected northward helps maintain the high densities of water in the North Atlantic and the vigorous formation of NADW. A reduction in deep water formation as a result of surface freshening reduces the poleward transport of salt and amplifies the initial perturbation; GCM simulations demonstrate that if the initial perturbation is sufficiently strong, then this large-scale advective feedback can drive the AMOC to another stable steady state in which the NADW formation and the overturning circulation are essentially turned off (e.g. Kuhlbrodt *et al.* 2007). The second set of local feedbacks involves the formation of NADW through deep convection, which homogenizes the water column into a convectively neutral state and transports relatively freshwater to depth. If convection is reduced due to surface freshening, then the reduced sinking flux of freshwater can amplify the initial surface

freshening and (for a sufficiently strong perturbation) shut off convection all together (e.g. [Rahmstorf 2001](#)). It is a generic result that the stability properties of nonlinear systems are affected by the presence of noise and environmental fluctuations that affect the AMOC are ubiquitous (e.g. [Weaver *et al.* 1999](#)). The effects of climate noise on transitions between the AMOC regimes will be considered in this section.

The simplest model that captures the AMOC bistability associated with large-scale advective feedbacks is that introduced by [Stommel \(1961\)](#), in which the circulation is associated with the exchange of heat and salt between two well-mixed boxes (representing the high- and low-latitude ocean in a single hemisphere) forced by specified freshwater fluxes and temperature relaxation to an externally specified difference. Denoting by ΔT and ΔS the interbox temperature and salinity differences, respectively, the respective tendencies can be expressed as

$$\frac{d}{dt}\Delta T = -q(\Delta\rho, t)\Delta T + \Gamma(\Delta T_0 - \Delta T), \quad (3.1)$$

$$\frac{d}{dt}\Delta S = -q(\Delta\rho, t)\Delta S + F(t), \quad (3.2)$$

where ΔT_0 is the externally imposed interbox temperature difference to which the system relaxes on a time scale of Γ^{-1} and F is the imposed freshwater forcing. The interbox exchange, q , is assumed to depend on the interbox density difference $\Delta\rho = \alpha_S\Delta S - \alpha_T\Delta T$ (α_S and α_T are, respectively, the haline and thermal expansivities of seawater) as $q(\Delta\rho, t) = \beta(t) + f(|\Delta\rho|)$, where $\beta(t)$ is a ‘diffusive’ exchange (including interbox fluxes mediated by gyre circulations; [Monahan 2002c](#)) and the function $f(\Delta\rho)$ models advective exchange by the overturning circulation (e.g. [Stommel 1961](#); [Cessi 1994](#)). As in [Monahan \(2002a\)](#), we focus on the model $f(|\Delta\rho|) = c|\Delta\rho|$; the following results are not qualitatively sensitive to this parametrization.

This already highly idealized system can be further simplified by taking the temperature relaxation time scale to be much faster than the interbox exchange time scale (a reasonable approximation), so to leading order $\Delta T \simeq \Delta T_0$ and we obtain a differential equation in ΔS alone. The influence of high-frequency fluctuations on the overturning will be modelled by taking $\beta(t)$ and $F(t)$ each to be the sum of a fixed mean and random fluctuations. Non-dimensionalizing the resulting equation as in [Monahan \(2002a\)](#), we obtain the stochastic differential equation (SDE)

$$\frac{d}{dt}y = -(b_0 + |1 - y|)y - y \circ \eta + \mu + \xi, \quad (3.3)$$

where y is the non-dimensional salinity difference; b_0 and η are, respectively, the mean and fluctuations of the non-dimensional diffusive exchange parameter; and μ and ξ are, respectively, the mean and fluctuations of the freshwater flux. The open circle indicates that, for η modelled as white noise, the SDE is interpreted in the Stratonovich sense (i.e. as the white noise approximation to an autocorrelated process; [Gardiner 1997](#); [Penland 2003](#)). Such highly idealized models of the effects of ‘weather’ forcing on the AMOC have been considered in a number of studies (e.g. [Stommel & Young 1993](#); [Cessi 1994](#); [Bryan & Hansen 1995](#);

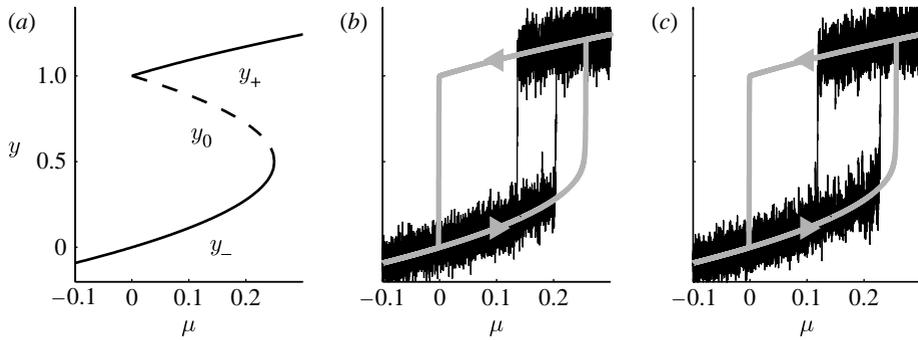


Figure 1. (a) Bifurcation structure of the AMOC box model equation (3.3) with $\eta = \xi = 0$ and $b_0 = 0$. The solid (dashed) lines are stable (unstable) solution branches. (b) Grey curve, hysteresis loop traced out by deterministic model as freshwater forcing μ is increased from -0.1 to 0.3 and then back to -0.1 . Black curve, realization of hysteresis curve for the stochastic model with $\eta = \sigma_1 \dot{W}_1$, $\xi = \sigma_2 \dot{W}_2$, where $\sigma_1 = \sigma_2 = 0.075$ and \dot{W}_i are the independent white noise processes. (c) As in (b), for a second realization of the stochastic model. With a very high probability, the stochastic hysteresis loops are smaller than the deterministic ones.

Lohmann & Schneider 1999; Timmermann & Lohmann 2000; Vélez-Belichí *et al.* 2001; Monahan 2002*a, b*; Monahan *et al.* 2002; Kleinen *et al.* 2002).

A plot of steady-state solutions of equation (3.3) for $b_0 = 0$ in the absence of fluctuations ($\eta = \xi = 0$) is given in figure 1*a*. For a range of values of the freshwater forcing parameter $0 \leq \mu \leq 0.25$, the idealized model admits three steady states. Two of these steady states are stable, y_+ and y_- , with weak (strong) overturning circulations and strong (weak) interbox density differences, respectively. The third steady state y_0 is unstable. This interval of bistability is bounded by fold bifurcations, beyond which only a single steady-state solution exists. If the parameter μ is increased from below the lower bifurcation point to above the upper bifurcation point and then decreased again to below the lower bifurcation, the hysteresis loop displayed in figure 1*b, c* is traced out. Because the steady states are equilibrium solutions, deterministic transitions between solution branches for an evolving freshwater forcing $\mu(t)$ will generally occur somewhat beyond the bifurcation point; that is, there will be a slight overshoot (e.g. Berglund & Gentz 2006). The fact that three-dimensional coupled ocean–atmosphere models produce analogous hysteresis structures (e.g. Weaver & Hughes 1994; Rahmstorf *et al.* 2005; Kuhlbrodt *et al.* 2007) suggests that this AMOC model captures the essential physics of the advective feedback bistability (although its predictions cannot be expected to be quantitatively meaningful).

In the absence of climate noise, the AMOC regime transitions can only occur if μ is moved beyond one of the fold bifurcations. However, in the presence of noise, spontaneous transitions between regimes can occur within the region of bistability; if the noise is unbounded (as with Gaussian fluctuations), then transitions are possible everywhere that both states exist. For η and ξ white noise processes, the mean transition time from y_- to y_+ can be computed analytically (e.g. Cessi 1994; Monahan 2002*a*) and takes the form $\tau(y_- \rightarrow y_+) \sim \exp(-V(y_-, y_0)/\Sigma^2)$, where Σ is a measure of the noise level and $V(y_-, y_0)$ is a measure of the ‘potential barrier’ the system must overcome to pass from y_- to y_+ (an analogous expression holds for the reverse transition). These transition rates are highly sensitive to the

strength of the noise forcing; a small change in Σ can change τ by orders of magnitude. If the transition time out of an AMOC regime is longer than any physically meaningful time scale, then spontaneous regime transitions will occur with vanishingly small probability (the limit considered by Bryan & Hansen (1995)). If, on the other hand, the residence times of both regimes are smaller than the longest physically meaningful time scales, the AMOC will pass back and forth between regimes, exploring thoroughly the available set of states. In this case, the signature of multiple regimes will be multimodality of the stationary (long-term equilibrium) pdf. For η and ξ white noise processes, the stationary pdf of the process governed by equation (3.3) can be determined analytically (Cessi 1994; Timmermann & Lohmann 2000; Monahan 2002a). Finally, if the noise strength is so large that typical excursions of $y(t)$ are much larger than the separation between y_- and y_+ , then the system will not feel the presence of the different deterministic equilibria and the two peaks of the stationary pdf will not be well separated (the limit considered by Stommel & Young (1993)); this high-noise case is not relevant to the AMOC.

If the intensity of the climate noise is independent of the state of the AMOC (i.e. if the noise is *additive*), then the primary effect of stochastic fluctuations is inducing transitions between the AMOC regimes (or exciting oscillations, as discussed in §2). However, if the intensity of one or more of the noise processes is dependent on the state of the system (i.e. if the noise is *multiplicative*), then the noise itself can alter qualitative aspects of the dynamics (e.g. Penland 2003). In equation (3.3), the fluctuations in diffusive exchange η enter multiplicatively (when η is taken to be white noise), and their intensity has an effect on the multimodality of the stationary AMOC pdf. As the intensity of η increases, the range of freshwater forcings μ over which the pdf is bimodal shifts and contracts, eventually vanishing (Timmermann & Lohmann 2000; Monahan 2002a). The domain of bistability is also altered if η and ξ are correlated (as they might be expected to be physically; Monahan 2002b). In this way, the structure of the AMOC regimes (rather than just their occupation statistics) is influenced by the stochastic climate forcing.

Shifts in the domain of bimodality produced by multiplicative noise persist when η is allowed to have a non-zero autocorrelation time (i.e. to be red noise). In this situation, the dynamics of the vector (y, η) is governed by a two-dimensional SDE with a stationary pdf that is multimodal outside of the range of freshwater forcing values μ for which the deterministic part of the dynamics has multiple equilibria (Monahan 2002b; Monahan *et al.* 2002; Timmermann & Lohmann 2000). Bimodality can occur in the absence of multiple deterministic equilibria when there is a neighbourhood of the (y, η) state space without a fixed point but with a local minimum of the magnitude of the deterministic tendency. Passing through such a region, the system slows down but does not come to a halt. Random fluctuations can drive the system from the neighbourhood of the fixed point into this ‘sluggish’ region, where it slows down and lingers, building up probability mass; the resulting pdf can be bimodal (Monahan 2002b).

For moderate values of the noise intensity, the residence times of the two model regimes generally differ by orders of magnitude. It follows that, for such noise levels, the probability of being in one regime is orders of magnitude greater than that of being in the other, so while the pdf is technically bimodal it is effectively unimodal. This phenomenon and its consequences were discussed in

Monahan (2002*a,b*), where it was referred to as regime stabilization by noise. One particularly significant consequence is that if the freshwater forcing approaches the fold bifurcation where the current AMOC regime vanishes, then with high probability a transition to the other regime will occur before the bifurcation point is reached (figure 1*b,c*). The point at which the transition occurs is random, with a mean value that depends on both the noise level and the rate at which μ is changing (e.g. Berglund & Gentz 2006). This effect is seen in both idealized models (Monahan 2002*a*) and more comprehensive coupled atmosphere–ocean models (Wang *et al.* 1999; Knutti & Stocker 2002). These transitions become less predictable as the bifurcation point is approached (Knutti & Stocker 2002). In the presence of climate noise, the AMOC regime shifts become less predictable as they become more likely.

The response of the modelled AMOC to climate noise does provide a potential ‘early warning system’ for regime transitions (Kleinen *et al.* 2002; Held & Kleinen 2004; Lenton *et al.* 2008). As the bifurcation point is approached, the real part of at least one eigenvalue of the dynamics linearized around the equilibrium state approaches zero (as at the bifurcation point, stable and unstable steady states coalesce). It follows that there is at least one state-space direction along which the feedbacks driving the system towards the equilibrium state weaken. As these feedbacks become weaker, perturbations excited along this weakly damped direction acquire larger amplitude and return more slowly to the equilibrium state. In consequence, the autocorrelation time scale and the variance of AMOC fluctuations both increase. Increasing trends in either of these statistics could herald the approach of a bifurcation point (as Carpenter & Brock (2006) have also noted in an ecological context), and these trends could potentially be measured using operational AMOC monitoring networks. The practical use of such an early warning system would depend on the ability to statistically distinguish anthropogenic trends from variability intrinsic to the climate system (Lenton *et al.* 2008).

The influence of fluctuating climate forcing on AMOC regime dynamics in more sophisticated ocean models indicates that the conclusions drawn from box models are qualitatively robust. In the zonally averaged two-dimensional ocean model used to study the response of centennial-scale AMOC variability to fluctuating freshwater forcing, Mysak *et al.* (1993) found noise-induced transitions between three distinct AMOC regimes in the limit of large horizontal diffusivity and small vertical diffusivity. Eyink (2005) obtained analytical solutions for the stationary pdf of the AMOC in a similar model. The deterministic equilibria and response to stochastic forcing in this model are broadly consistent with the box model results presented above, despite the considerably greater sophistication of the model. Furthermore, the AMOC transition early warning system originally characterized in a box model by Kleinen *et al.* (2002) was shown to be characteristic of a coupled ocean–atmosphere model of intermediate complexity in Held & Kleinen (2004).

The discussion of AMOC regimes has so far focused on those associated with large-scale advective feedbacks. In the absence of stochastic forcing, local convective feedbacks lead to either multimodal or oscillatory behaviour in models (e.g. Weaver *et al.* 1993; Cessi 1996; Rahmstorf 2001). For the former case, the effects of climate noise are broadly the same as those associated with the advective feedback multiple equilibria (Kuhlbrodt *et al.* 2001; Kuhlbrodt & Monahan 2003).

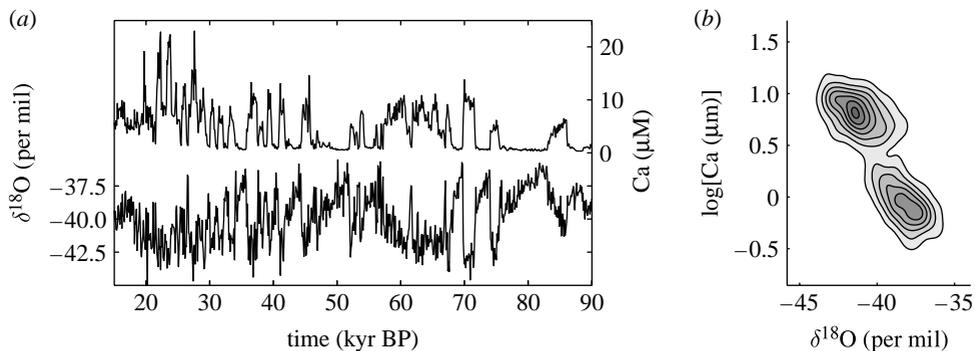


Figure 2. Oxygen isotope and Ca concentration data from the GRIP ice core. The $\delta^{18}\text{O}$ record measures the excess of O^{18} over O^{16} in the ice and is a measure of high-latitude temperature (higher $\delta^{18}\text{O}$ corresponding to warmer temperatures), while the Ca record is a measure of atmospheric dustiness (Fuhrer *et al.* 1999). (a) $\delta^{18}\text{O}$ and Ca concentration from 15 000 to 90 000 years BP, linearly interpolated to a nominally uniform temporal resolution of 100 years. (b) Joint pdf of $\delta^{18}\text{O}$ and log Ca.

For the latter case, stochastic forcing can modify the character of the oscillations and the parameter range over which they occur (Weaver *et al.* 1993; Cessi 1996), as well as driving the AMOC between different oscillating regimes (Aeberhardt *et al.* 2000). Furthermore, modelled oceanic diffusive processes have been shown to produce millennial-scale oscillatory responses with time scales strongly dependent on fluctuating freshwater forcing (e.g. Weaver & Hughes 1994).

4. Stochastic resonance and Dansgaard–Oeschger events

Evidence from palaeoclimate records, particularly high-latitude ice and ocean sediment cores, demonstrates that the climate of the last glacial period was characterized by a succession of abrupt shifts between relatively cold (stadial) and relatively warm (interstadial) states (e.g. Rahmstorf 2002). These transitions, known as Dansgaard–Oeschger (DO) events, are evident in the records of ice oxygen isotopic composition ($\delta^{18}\text{O}$; a measure of high-latitude temperature) and calcium concentration (a measure of atmospheric dustiness) from the Greenland Ice Core Project (GRIP) presented in figure 2. In particular, the joint distribution of $\delta^{18}\text{O}$ with the logarithm of the Ca concentration is manifestly bimodal with clearly separated stadial and interstadial regimes (Fuhrer *et al.* 1999). It is evident from figure 2 that while DO events occur with millennial time scales, they do not simply reflect a regular sinusoidal oscillation of the climate system.

While feedbacks in many different components of the climate system are involved in DO events (as external periodic forcing might also be; see below), the AMOC is believed to play a central role in stadial/interstadial transitions as a result of (i) the importance of the AMOC for the transport of heat to the high-latitude North Atlantic, and (ii) the potentially nonlinear response of the AMOC to buoyancy forcing (e.g. Clark *et al.* 2002; Rahmstorf 2002). Palaeoclimate data indicate that the climate system is much more variable during glacial periods

than during interglacials, and provide evidence of changes in the location and rate of NADW formation associated with stadial/interstadial transitions (Clark *et al.* 2002). Modelling studies suggest that the stability properties of the AMOC are also considerably different between glacial and interglacial periods (e.g. Ganopolski & Rahmstorf 2001; Schmittner *et al.* 2002): during glacial periods, a stadial circulation state with NADW formation in the subpolar North Atlantic is found to be stable, while an interstadial state with NADW formation in the Nordic Seas is unstable but easily excited from the stadial state by freshwater perturbations to the North Atlantic. While the interstadial state is not steady, the trajectory of the system is relatively slow in its immediate neighbourhood. This picture is consistent with the observed evolution of DO events: a rapid transition from stadial to interstadial is followed by a gradual relaxation of the AMOC towards the stadial state with a final rapid shift. Note that simple models of the AMOC discussed in §3 demonstrate that it is possible for the pdf of the stochastic system to be bimodal (as seen in the palaeoclimate records; figure 2) even when the deterministic component of the system has a single fixed point, if the deterministic tendency takes a local minimum in some neighbourhood into which the system is easily excited. Evidence for a third circulation state with essentially no NADW production (the so-called ‘Heinrich mode’) is also found in palaeoclimate records and in climate models (e.g. Rahmstorf 2002; Schmittner *et al.* 2002). High-latitude North Atlantic temperatures during a Heinrich mode are essentially the same as those during a stadial period, consistent with both states being associated with a dramatic reduction in the oceanic transport of heat to North Atlantic high latitudes.

Analyses of North Atlantic palaeoclimate records (both glacial and deep sea) of the last glacial period suggest the presence of an approximately 1500-year periodic signal associated with the sequence of DO events (e.g. Mayewski *et al.* 1997). The existence of a well-defined spectral peak in the time series, however, depends on both the dataset (e.g. Ditlevsen *et al.* 2005) and the time period (e.g. Schulz 2002) under consideration, and has been suggested to be a spurious signal resulting from aliasing of the annual cycle (Wunsch 2000). Furthermore, there is evidence (still controversial, as discussed below) that DO events do not occur with strictly regular periodicity, but are separated by time intervals that are approximately integer multiples of 1470 years (Alley *et al.* 2001; Schulz 2002; Rahmstorf 2003). Such behaviour is the hallmark of the phenomenon of *stochastic resonance* (SR) in which the addition of noise to a periodic sub-threshold signal results in approximately periodic crossings of some particular threshold. On occasion, one or more crossings will be missed, so intervals between successive crossings will cluster together around integer multiples of the period of the forcing; the distribution of these crossing times will decay exponentially (Alley *et al.* 2001). Stochastic resonance was originally introduced as a model for glacial/interglacial transitions in response to Milankovitch forcing (Benzi *et al.* 1982), and, although it does not appear to be relevant in this original context, SR has since been found to be characteristic of a broad range of physical and biological systems (e.g. Gammaitoni *et al.* 1998).

Vélez-Belichí *et al.* (2001) used a stochastic box model (such as in §3) to demonstrate SR in the AMOC in response to periodic forcing; although this study focused on Milankovitch forcings rather than the millennial time scales characteristic of DO events, it demonstrated that SR occurs over a broad range

of driving periods. Stochastic resonance on millennial time scales was demonstrated by Ganopolski & Rahmstorf (2002) in a more sophisticated coupled atmosphere–ocean model forced by boundary conditions appropriate for the last glacial period. In this study, a small sinusoidal freshwater perturbation with a period of 1470 years was applied to the North Atlantic, along with stochastic freshwater fluxes (with amplitudes comparable to present-day interannual variability). In response, the simulated AMOC displayed SR, alternating between stadial and interstadial circulation states with a distribution of transition times that compared favourably with those of DO events in the GRIP ice core. Stochastic resonance could be achieved with realistic noise levels because the modelled interstadial state is easily excited from the stadial state by freshwater perturbations to the North Atlantic (Ganopolski & Rahmstorf 2001; Schmittner *et al.* 2002). In Ganopolski & Rahmstorf (2002), the 1470-year periodic forcing was some external perturbation of unknown provenance; using the same model, Braun *et al.* (2005) suggested that the forcing may in fact arise through the superposition of 87- and 210-year Gleissberg and DeVries solar cycles. When forced by a linear combination of North Atlantic freshwater forcings with these periodicities (or a more realistic modulation of the 11-year solar cycle by the Gleissberg cycle), the model responds with a 1470-year almost-periodic alternation between stadial and interstadial states (for certain forcing parameter ranges). That the modelled AMOC should respond on millennial time scales to this shorter time-scale forcing was attributed to a combination of strongly nonlinear dynamics and the long intrinsic AMOC adjustment time scales.

Using an idealized coupled atmosphere–ocean–sea-ice model, Timmermann *et al.* (2003) suggested a variation on the SR mechanism for driving cycles of DO events. Instead of being driven by an external periodic signal, DO events in this model occur through a combination of noise and a periodic limit cycle internal to the climate system itself in a phenomenon known as *coherence resonance* (CR) or *autonomous* SR (also discussed in Ganopolski & Rahmstorf 2002). An essential (and testable) difference between SR and CR is that, in the former, the phase of the external driving signal and the resulting transitions is fixed, while, in the latter, the phase of the oscillation can drift as a result of internal climate system interactions.

Stochastic resonance is a meaningful candidate mechanism for driving DO event cycles only to the extent that the distribution of observed inter-transition times clusters around integer multiples of a single (1470-year) time scale, with transitions that are phase-locked to this external periodic forcing. An exponential distribution of transition times without this clustering would be suggestive of stadial–interstadial transitions being driven by climate noise alone, without a periodic forcing (e.g. Ditlevsen *et al.* 2007), while a looser clustering of transition times without locking to a periodic signal of fixed phase would be suggestive of CR. Distinguishing between these alternatives is a statistical problem complicated by (i) the difficulty of defining precisely when a transition has occurred, (ii) problems with ice core chronology, and (iii) the diversity of null hypotheses against which the data can be compared. Time-series analyses of high-latitude palaeoclimate records by Roe & Steig (2004) and Ditlevsen *et al.* (2005, 2007) suggested that a statistical model with stadial/interstadial transitions driven by climate noise without a preferred periodic forcing provides

a better fit to the observations than a stochastically resonant model, particularly for the newly obtained NGRIP ice core (Ditlevsen *et al.* 2007). As well, the influence of solar variability on the onset of DO events has been questioned (Muscheler & Beer 2006). Owing to the numerous uncertainties involved in the reconstruction of past climates, SR remains a controversial mechanism for the pacing of DO cycles.

5. Conclusions

Variability of the oceanic MOC is an important component of variability in the climate system on time scales from decades through centuries to millennia. Modelling studies suggest that MOC variability on these ‘climate’ time scales may be strongly influenced by fluctuations in surface forcing on much shorter weather time scales. Such high-frequency forcing has typically entered models of the MOC through stochastic processes parametrizing unresolved atmospheric processes, representing fluctuations in surface buoyancy and (less often) mechanical fluxes. A significant gap in our understanding of the importance of stochastic forcing comes not through its role in external forcing but rather through the means that the effects of this external forcing are parametrized in ocean and climate models. The development of physically consistent representations of fluctuations in unresolved scales and their dependence upon resolved scales—that is, of stochastic subgrid-scale parametrizations—remains a significant physical and mathematical challenge (e.g. Palmer 2001). In particular, mechanical forcing provides the energy necessary to drive the thermohaline circulation both via direct mixing (through both wind- and tidally generated internal wave breaking) or through wind-driven upwelling in the Southern Ocean (e.g. Kuhlbrodt *et al.* 2007). A natural question arises as to whether or not the circulation in large-scale ocean models is sensitive to random fluctuations in mixing associated with the internal wave field, which is patchy and episodic.

Recent observations of MOC transport at 26.5° N made from the rapid climate change (RAPID) mooring array measure variability in the maximum meridional overturning, with a standard deviation of 5.6 Sv around the mean value of 18.7 Sv on sub-annual time scales from 2004 to 2005 (Cunningham *et al.* 2007). The existence of such variability has important consequences for predictability of the MOC, both because it can make detection of trends difficult and because these fluctuations themselves may influence the timing of transitions between circulation regimes. Weather is not simply noise superimposed upon climate: interactions across time scales are essential to the dynamics of the climate system. Probability theory and stochastic dynamics provide natural tools for investigating the connection between weather and climate; this paper has presented an overview of the insights into the variability of the MOC that have so far resulted from the application of these tools to this problem. The study of the interaction between weather and climate in the MOC is relatively young: as models and the observational record both improve, we are confident that this perspective will play an increasingly large role in understanding the dynamics of this fundamentally important component of the climate system.

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