Definition of Capacitance

- The **capacitance**, C, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the potential difference between the conductors

$$C \equiv \frac{Q}{\Delta V}$$

- The SI unit of capacitance is the **farad** (F)

Capacitance – Parallel Plates

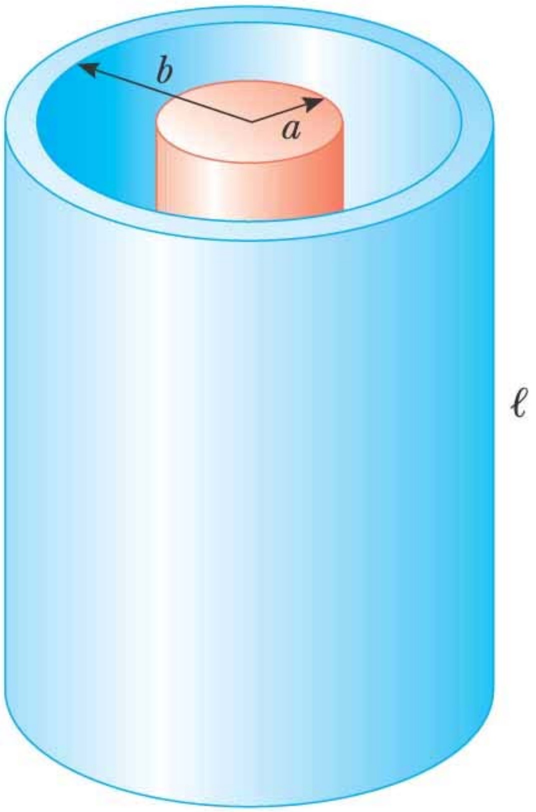
- The capacitance is proportional to the area of its plates and inversely proportional to the distance between the plates

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{Q}{Qd / \varepsilon_o A} = \frac{\varepsilon_o A}{d}$$

Capacitance of a Cylindrical Capacitor

- $\Delta V = -2k_e\lambda \ln (b/a)$
- $\lambda = Q/\ell$
- The capacitance is

$$C = \frac{Q}{\Delta V} = \frac{\ell}{2k_e \ln\left(b/a\right)}$$



(a)

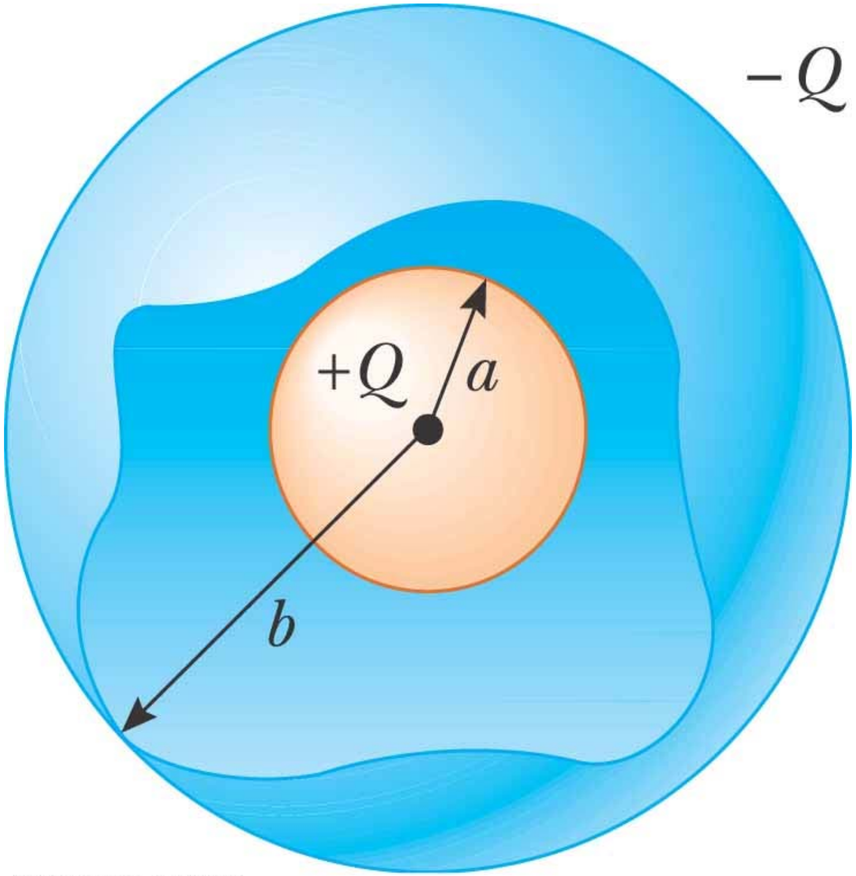
Capacitance of a Spherical Capacitor

- The potential difference will be

$$\Delta V = k_e Q\left(\frac{1}{b} - \frac{1}{a}\right)$$

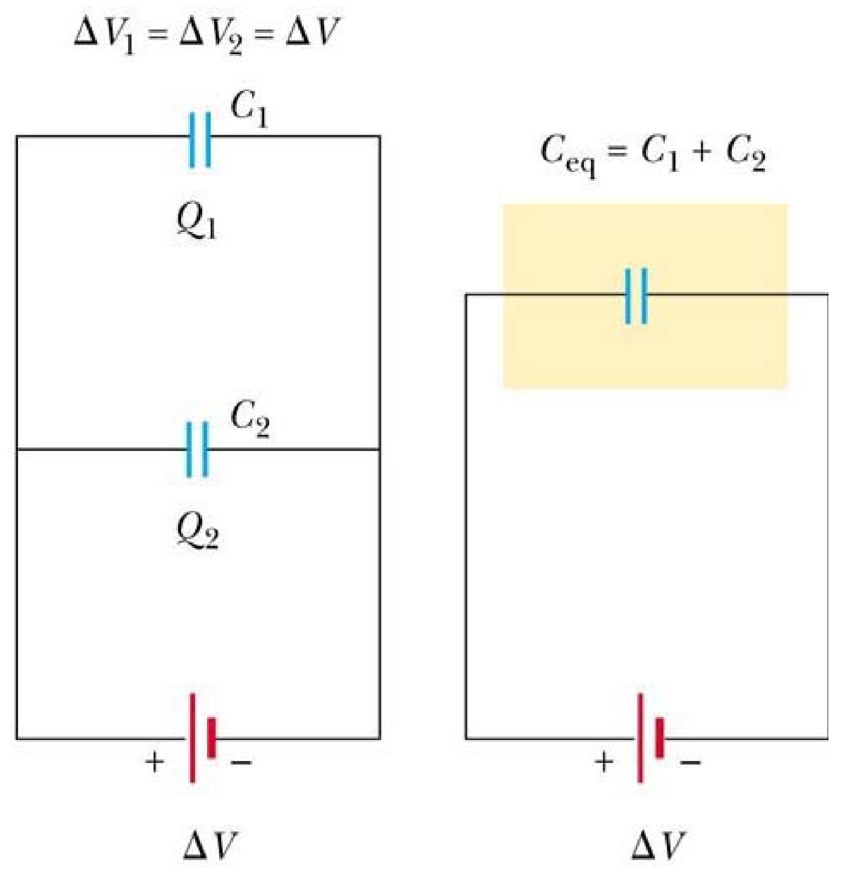
- The capacitance will be

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e\left(b-a\right)}$$



Capacitors in Parallel, 3

- The capacitors can be replaced with one capacitor with a capacitance of C_{eq}
  – The *equivalent capacitor* must have exactly the same external effect on the circuit as the original capacitors



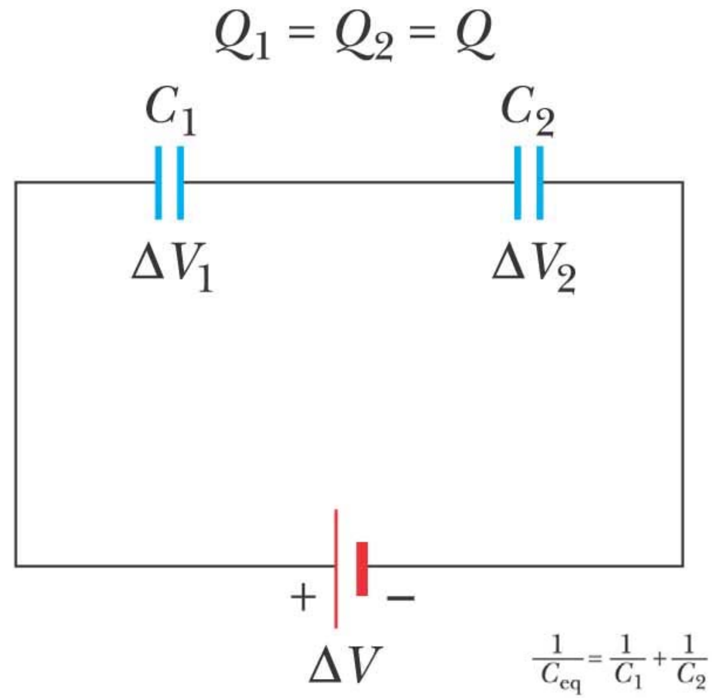
Capacitors in Parallel

- $C_{eq} = C_1 + C_2 + \mathrm{C}_3 + \ldots$
- The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors
  - Essentially, the areas are combined

Capacitors in Series

- An equivalent capacitor can be found that performs the same function as the series combination
- The charges are all the same

  $Q_1 = Q_2 = Q$



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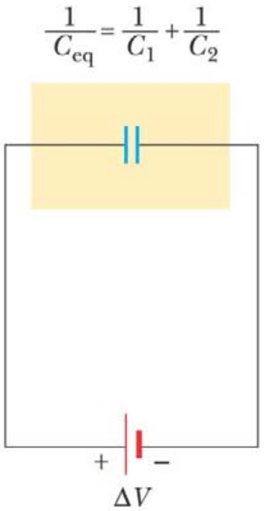

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Capacitors in Series

- The potential differences add up to the battery voltage

  $\Delta V_{tot} = \Delta V_1 + \Delta V_2 + \ldots$
- The equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

- The equivalent capacitance of a series combination is always less than any individual capacitor in the combination

Energy Stored in a Capacitor

- Assume the capacitor is being charged and, at some point, has a charge *q* on it
- The work needed to transfer a charge from one plate to the other is

$$dW = \Delta V dq = \frac{q}{C} dq$$

- The total work required is

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

Energy

- The work done in charging the capacitor appears as electric potential energy *U*:

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

- This applies to a capacitor of any geometry
- The energy stored increases as the charge increases and as the potential difference increases
- In practice, there is a maximum voltage before discharge occurs between the plates

Energy

- The energy can be considered to be stored in the electric field
- For a parallel-plate capacitor, the energy can be expressed in terms of the field as $U = \frac{1}{2}(\varepsilon_o A d)E^2$
- It can also be expressed in terms of the energy density (energy per unit volume) $u_E = \frac{1}{2}\varepsilon_o E^2$

Some Uses of Capacitors

- Defibrillators
  - When cardiac fibrillation occurs, the heart produces a rapid, irregular pattern of beats
  - A fast discharge of electrical energy through the heart can return the organ to its normal beat pattern
- In general, capacitors act as energy reservoirs that can be slowly charged and then discharged quickly to provide large amounts of energy in a short pulse

Capacitors with Dielectrics

- A *dielectric* is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance
  - Dielectrics include rubber, glass, and waxed paper
- With a dielectric, the capacitance becomes $C = \kappa C_o$
  - The capacitance increases by the factor κ when the dielectric completely fills the region between the plates
  - κ is the **dielectric constant** of the material

Dielectrics, cont

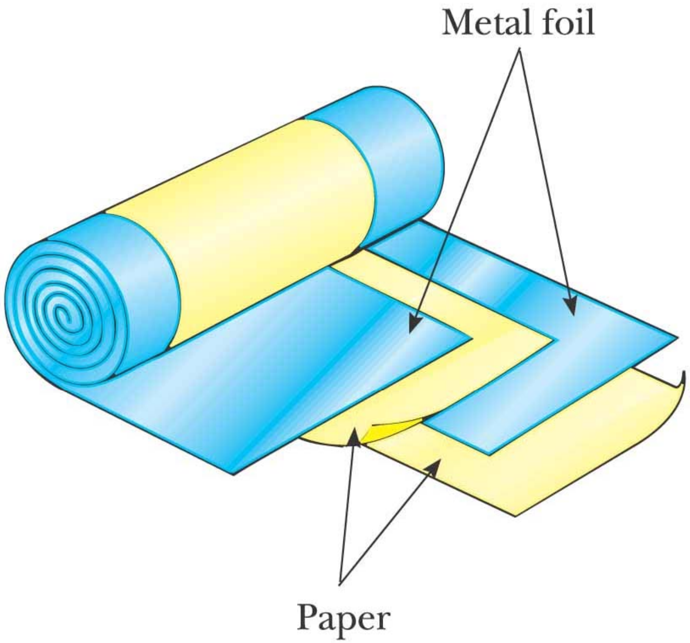
- For a parallel-plate capacitor, $C = \kappa\varepsilon_o(A/d)$
- In theory, *d* could be made very small to create a very large capacitance
- In practice, there is a limit to *d*
  - *d* is limited by the electric discharge that could occur though the dielectric medium separating the plates
- For a given *d*, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** of the material

Dielectrics, final

- Dielectrics provide the following advantages:
  - Increase in capacitance
  - Increase the maximum operating voltage
  - Possible mechanical support between the plates
    - This allows the plates to be close together without touching
    - This decreases d and increases C

Types of Capacitors – Tubular

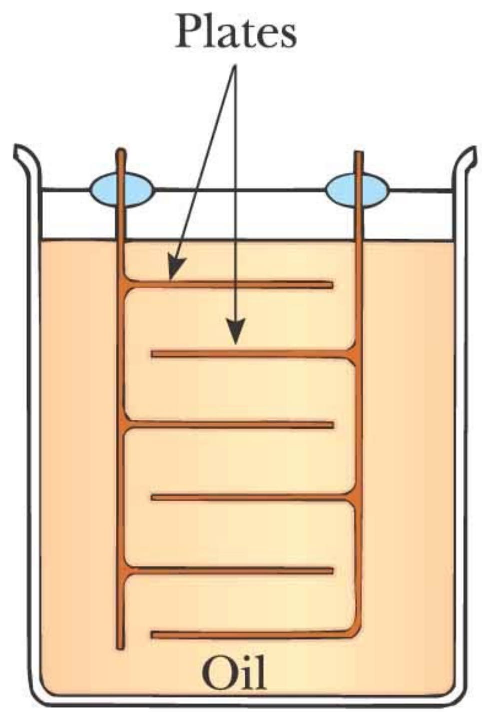
- Metallic foil may be interlaced with thin sheets of paraffin-impregnated paper or Mylar
- The layers are rolled into a cylinder to form a small package for the capacitor



(a)

Types of Capacitors – Oil Filled

- Common for high-voltage capacitors
- A number of interwoven metallic plates are immersed in silicon oil



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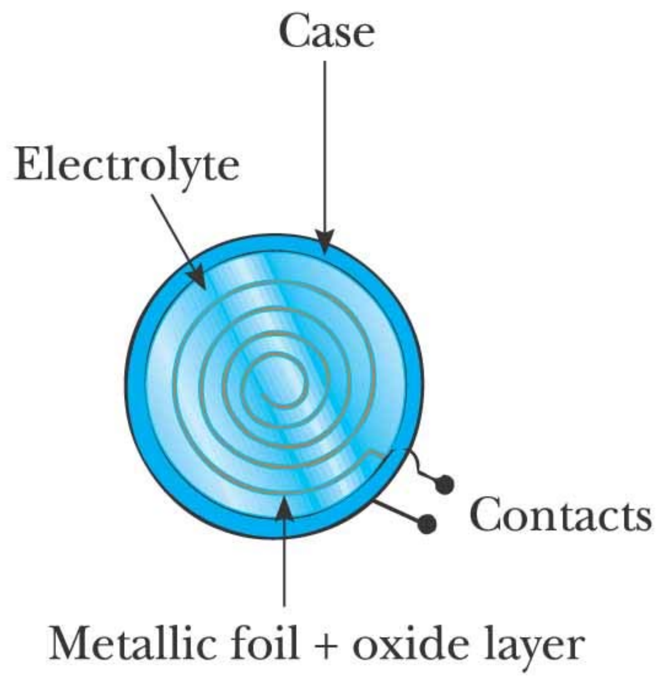
Types of Capacitors – Electrolytic

- Used to store large amounts of charge at relatively low voltages
- The electrolyte is a solution that conducts electricity by virtue of motion of ions contained in the solution



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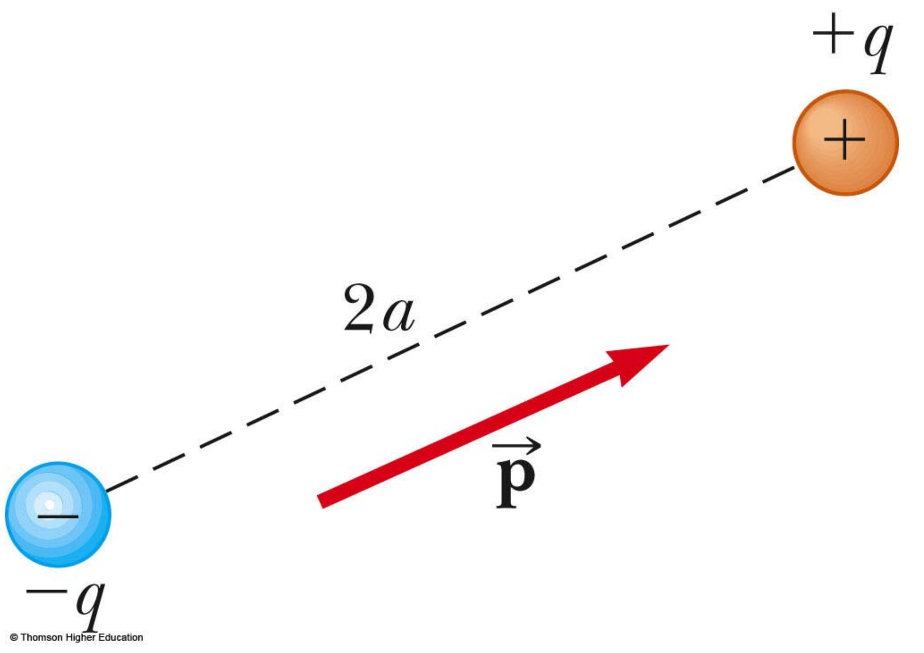
Electric Dipole

- An electric dipole consists of two charges of equal magnitude and opposite signs
- The charges are separated by 2*a*
- The **electric dipole moment** ($\vec{\mathbf{p}}$) is directed along the line joining the charges from –*q* to +*q*

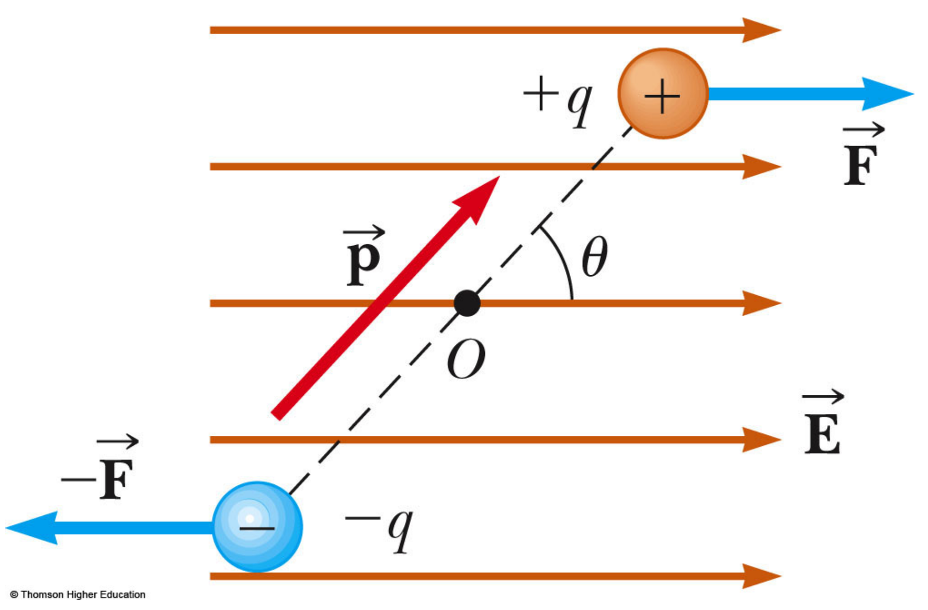


Electric Dipole, 2

- The electric dipole moment has a magnitude of $p \equiv 2aq$
- Assume the dipole is placed in a uniform external field, $\vec{\mathbf{E}}$
  - $\vec{\mathbf{E}}$ is external to the dipole; it is not the field produced by the dipole
- Assume the dipole makes an angle θ with the field

Electric Dipole, 3

- Each charge has a force of $F = Eq$ acting on it
- The net force on the dipole is zero
- The forces produce a net torque on the dipole



Electric Dipole, final

- The magnitude of the torque is:

  $\tau = 2Fa \sin\theta = pE \sin\theta$

- The torque can also be expressed as the cross product of the moment and the field:

  $\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$

- The potential energy can be expressed as a function of the orientation of the dipole with the field:

  $U_f - U_i = pE(\cos\theta_i - \cos\theta_f) \rightarrow$

  $U = \vec{\mathbf{p}} \cdot \vec{\mathbf{E}}$

  $U = -pE\cos\theta$