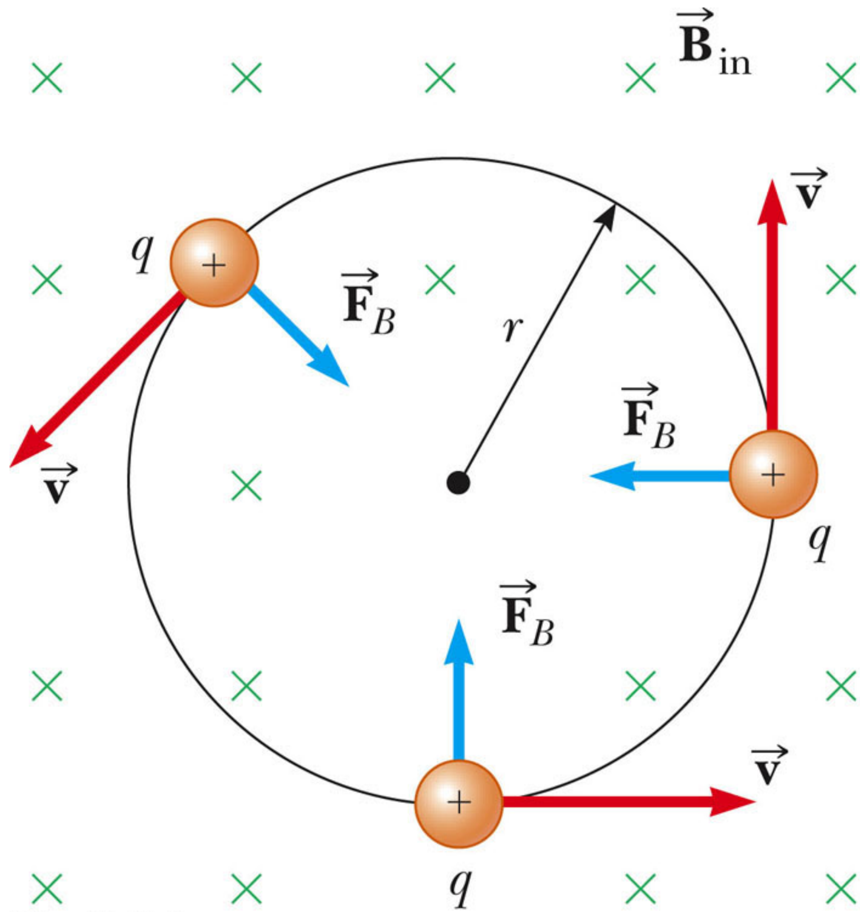
Charged Particle in a Magnetic Field

- Consider a particle moving in an external magnetic field with its velocity perpendicular to the field
- The force is always directed toward the center of the circular path
- The magnetic force causes a centripetal acceleration, changing the direction of the velocity of the particle
- Use the active figure to change the parameters of the particle and observe the motion



PLAY ACTIVE FIGURE

Force on a Charged Particle

- Equating the magnetic and centripetal forces:

$$F_B = qvB = \frac{mv^2}{r}$$

- Solving for r:

$$r = \frac{mv}{qB}$$

  – r is proportional to the linear momentum of the particle and inversely proportional to the magnetic field

More About Motion of Charged Particle

- The angular speed of the particle is

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

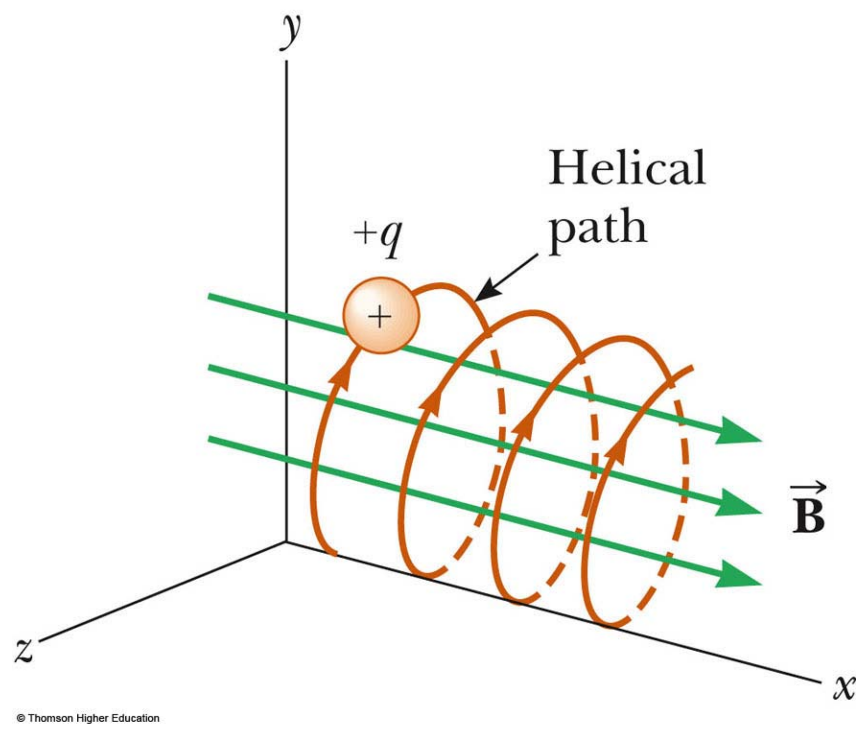
  - The angular speed, ω, is also referred to as the **cyclotron frequency**

- The period of the motion is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

Motion of a Particle, General

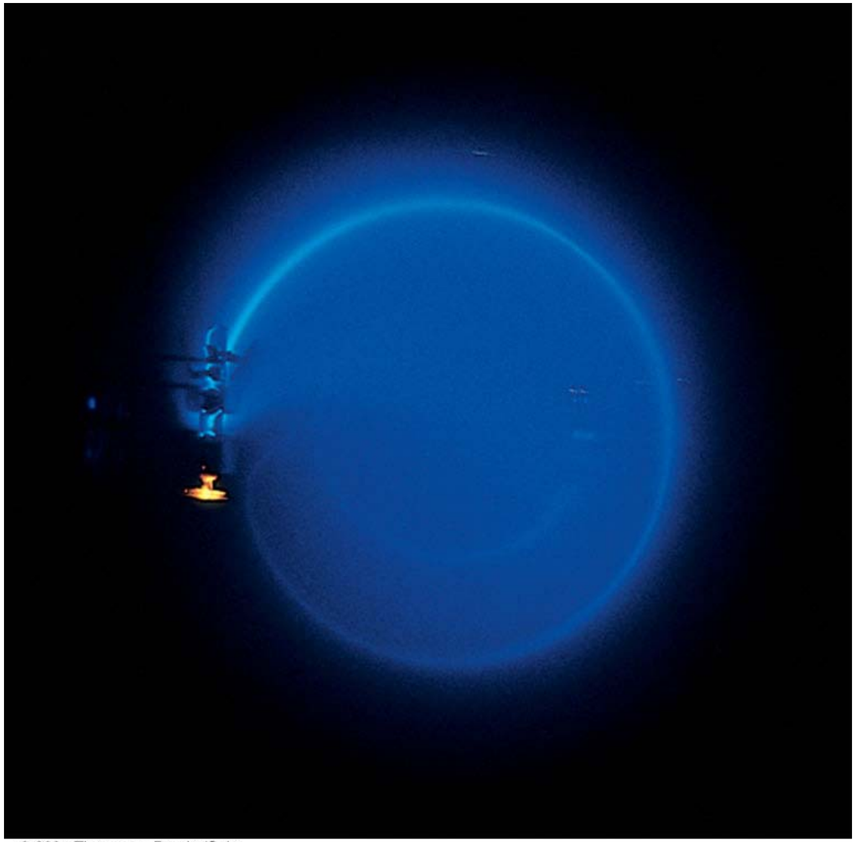
- If a charged particle moves in a magnetic field at some arbitrary angle with respect to the field, its path is a helix
- Same equations apply, with $v_{\perp} = \sqrt{v_y^2 + v_z^2}$
- Use the active figure to vary the initial velocity and observe the resulting motion



PLAY ACTIVE FIGURE

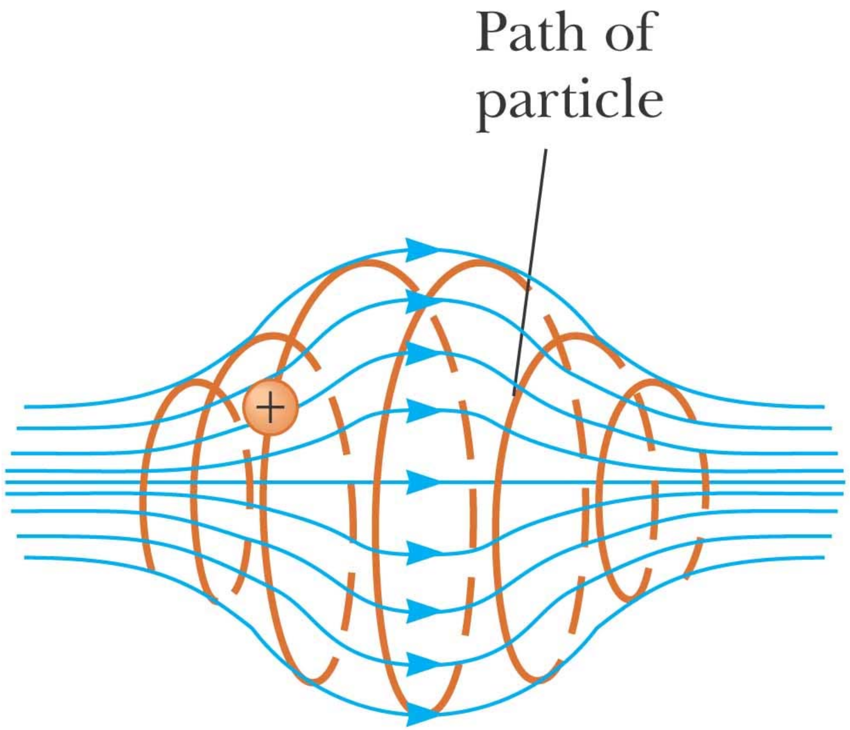
Bending of an Electron Beam

- Electrons are accelerated from rest through a potential difference
- The electrons travel in a curved path
- Conservation of energy will give *v*
- Other parameters can be found



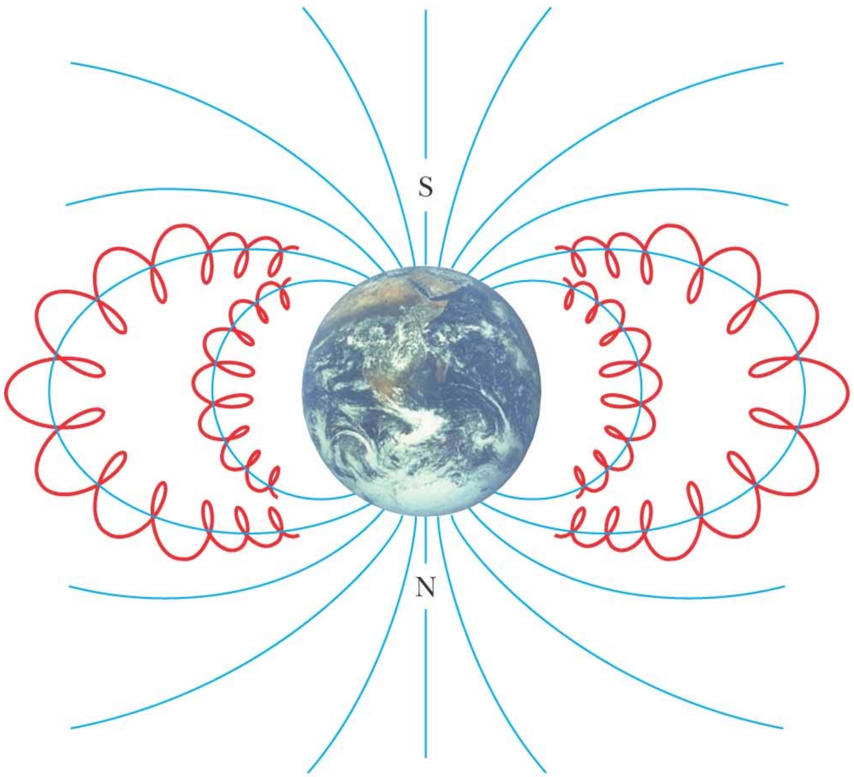
Particle in a Nonuniform Magnetic Field

- The motion is complex
- For example, the particles can oscillate back and forth between two positions
- This configuration is known as a *magnetic bottle*



Van Allen Radiation Belts

- The Van Allen radiation belts consist of charged particles surrounding the Earth in doughnut-shaped regions
- The particles are trapped by the Earth's magnetic field
- The particles spiral from pole to pole
  - May result in Auroras

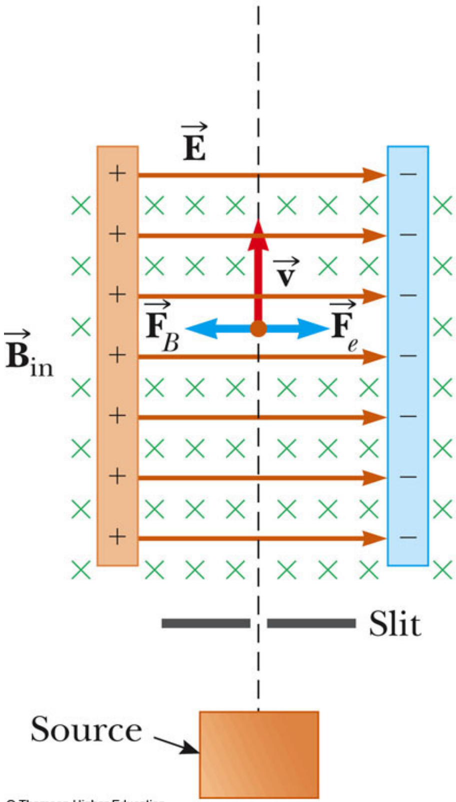


Charged Particles Moving in Electric and Magnetic Fields

- In many applications, charged particles will move in the presence of both magnetic and electric fields
- In that case, the total force is the sum of the forces due to the individual fields
- In general: $\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

Velocity Selector

- Used when all the particles need to move with the same velocity
- A uniform electric field is perpendicular to a uniform magnetic field
- Use the active figure to vary the fields to achieve the straight line motion



PLAY ACTIVE FIGURE

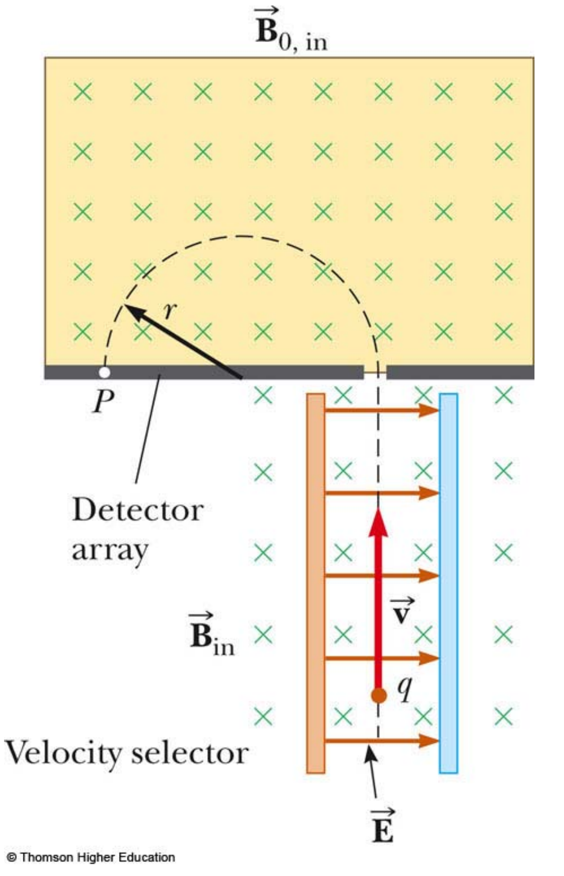
Velocity Selector, cont.

- When the force due to the electric field is equal but opposite to the force due to the magnetic field, the particle moves in a straight line
- This occurs for velocities of value

  $v = E / B$, found using qvB=qE

Mass Spectrometer

- A mass spectrometer separates ions according to their mass-to-charge ratio
- A beam of ions passes through a velocity selector and enters a second magnetic field
- Use the active figure to see where the particles strike the detector array



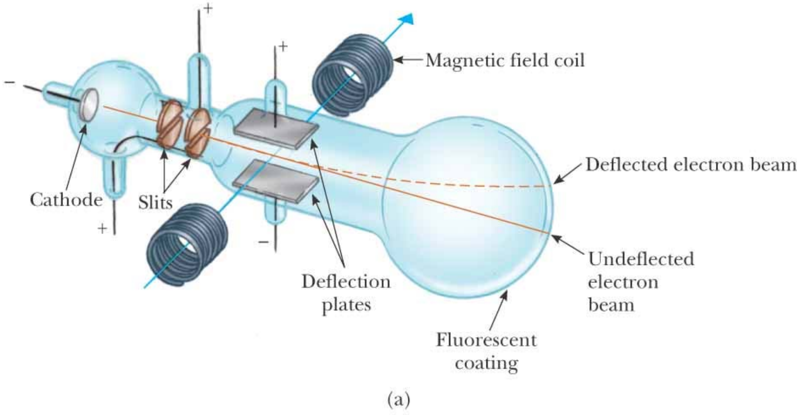
PLAY
ACTIVE FIGURE

Mass Spectrometer, cont.

- After entering the second magnetic field, the ions move in a semicircle of radius r before striking a detector at P
- If the ions are positively charged, they deflect to the left
- If the ions are negatively charged, they deflect to the right
- $mv^2/R=qvB$, therefore $m/q=RB/v$

Thomson's *e*/*m* Experiment

- Electrons are accelerated from the cathode
- They are deflected by electric and magnetic fields
- The beam of electrons strikes a fluorescent screen
- e/m was measured

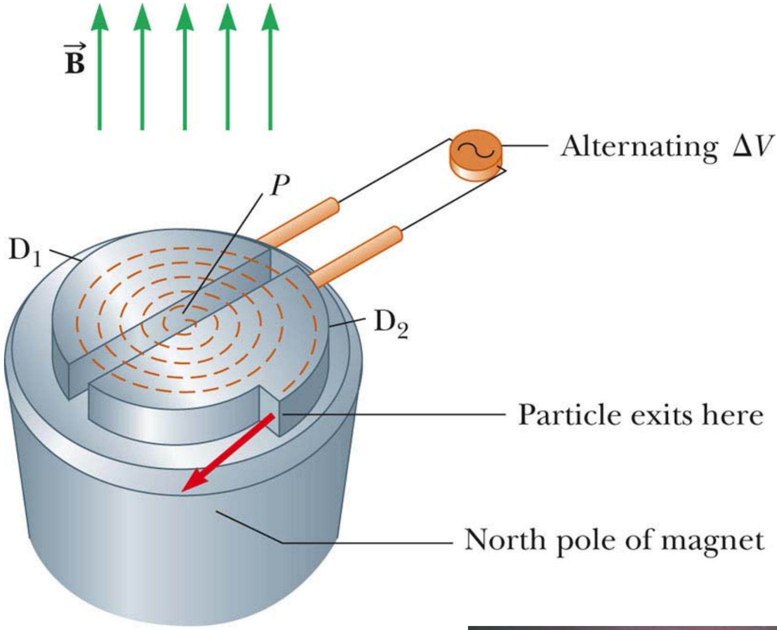


Cyclotron

- A **cyclotron** is a device that can accelerate charged particles to very high speeds
- The energetic particles produced are used to bombard atomic nuclei and thereby produce reactions
- TRIUMF at UBC is largest cyclotron in the world, it accelerates H^- ions

Cyclotron, 2

- D_1 and D_2 are called *dees* because of their shape
- A high frequency alternating potential is applied to the dees
- A uniform magnetic field is perpendicular to them



(a)


(b)

Cyclotron, final

- The cyclotron's operation is based on the fact that T is independent of the speed of the particles and of the radius of their path

$$K = \frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$$

- When the energy of the ions in a cyclotron exceeds about 20 MeV, relativistic effects come into play

TRIUMF

- 10^{15} particles/s, 200keV/gap, 500MeV max
- 23MHz gap frequency, gap=20cm
- 1500 turns in 1/3000 s, path length 45 km
- B up to 5600 G, current for B =18,500 A
- 6 pinwheel magnets, 4000 tonnes, d=60ft
- $27 million budget, $500 million to date